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SUBSTITUTION TEST BETWEEN INFLATION AND UNEMPLOYMENT IN IRAN: AN APPLICATION OF KALMAN FILTER

Abstract. *The present paper addresses substitution relation between inflation and unemployment in Iran, drawing on Kalman filter for 1971–2009. To achieve this end, four scenarios were designed. The results of these scenarios indicated that of the four only the third scenario models the variable coefficient in a random walk way and in which the coefficients are significant and compatible with theoretical principles. Moreover, the results implied a reverse relationship between inflation and unemployment in Iran. Recommendations emergent from this study include adopting policies which would provoke supply and also reducing inflationary expectations.*
Keywords: *Inflation Rate, Unemployment Rate, Kalman Filter, Iran.*

JEL Classification: C13, E24, E31, J64.

1. Introduction

Decreasing inflation rate and unemployment is among the most important objectives followed by economic theoreticians and policy-makers. Notwithstanding their social effects, inflation and unemployment can lead to the loss of an amount of economic resources and capacities. Fisher (1926) first demonstrated the indirect relation between inflation and unemployment-whose direction was from inflation to unemployment-using the data related to the US between 1915 and 1925. Phillips (1958), followed suit in his paper published and addressed the "The Relation between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom". He concluded that there is an indirect relationship between the two variables in long term.

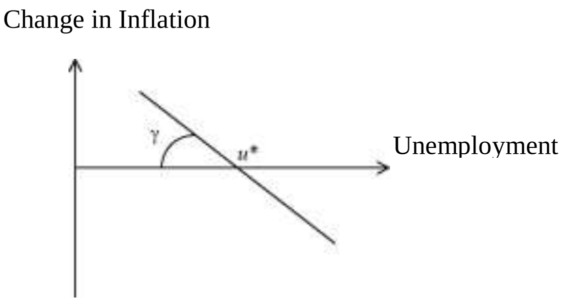

Figure 1. Linear Phillips Curve

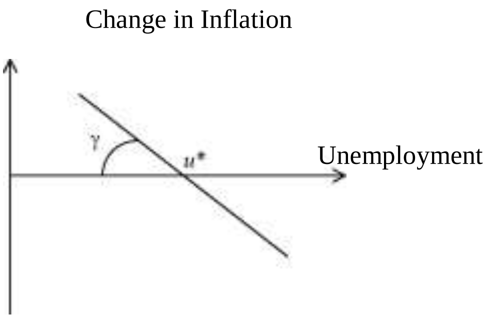

Figure 2. Convex Phillips Curve

Figure 1 depicts a linear Phillips curve. The trade-off parameter, γ, is constant. In Figure 2, γ increases with the unemployment gap, in a capacity constraint interpretation (Note that a "positive unemployment gap" is defined here as unemployment being below $u*$, the NAIRU). From an inflation asymmetry point of view, however, it can also be said that $\gamma$is bigger, or that the curve becomes more vertical, when inflation increases (Aubyn, 2000).

Showing the relationship between inflation and economic growth in Phillips curve is done on the basis of Okun's (1962) law. Okun investigated the relationship between unemployment rate and economic growth rate in 1960-1994 time spans in the US. Okun's law is an empirical relationship which expresses that for each 2.5 percent of growth beyond the normal extent in the real GDP (which keeps up for one year); unemployment would decrease by 1 percent. According to this rule, there is a positive relationship between inflation rate and economic growth rate. In a study in Britain between 1862 and 1957, Lipsey (1960) confirmed Phillips' findings. He found a nonlinear relationship between the change in wages and change in unemployment rate. According to Keynes, workers suffer from money illusion. Given this illusion, therefore, the wages do not increase as the prices rise: as the real wages decrease, hiring through agencies increase and so do employment and production. Consequently, there is a negative relationship between inflation and unemployment but a positive relationship between inflation and production (Da Silva, 2013).

Friedman's (1968) and Phelps' (1968) analyses of Phillips curve-which are based on comparative expectations-show that, in short term, the expected inflation of labour is lower than real inflation. Phillips curve, therefore, has a negative slope and moves with changes in expectations. In long term, however, the expectation of labour is fully formed and the real inflation becomes equal to expectations. As a result, wages rise in tandem with prices, overall supply becomes vertical and so does Phillips curve. This means there is not, in long term, a relationship between inflation and unemployment and between production and employment. According to the classic theories, prices and wages are totally flexible, and information exists in its fullest form and expectations are formed in a rational way. In this approach, market equality takes place fast and the overall supply curve is vertical. As a result, Phillips curve is vertical too and there is not a relationship between inflation, unemployment and production.

In Iran, high unemployment rate and inflation have always brought about concerns. On one hand, excluding some portion of the labour, unemployment diminishes the amount of production; on the other, it leads to a decrease in skill and efficiency of the labour by separating people from work environment. Inflation decreases shopping power of groups with stable income by widening class gap and income difference-which disturbs resource allocation consequently. Recognizing and explaining the relationship between inflation and unemployment in the country's economy, therefore, occupy an important position in economic decision-taking. Given the importance of this issue and that inflation and unemployment are among the chronic phenomena in Iran, the objective of the current project is to empirically investigate the relationship between inflation and unemployment. To achieve this end, the paper has been divided into five sections. At first, the related literature is examined in theoretical and empirical terms. In section 3, the research methodology is introduced. Section 4 presents the findings of the research, and the final section presents the conclusions.

2. The Related Literature

2.1 Theoretical Analyses

Most of the theoretical analyses are devoted to the relationship between inflation and unemployment as are discussed in Phillips curve. Schools of economics have analysed the relationship, given the intellectual infrastructure and theoretical principles. Fisher (1926) was the first one to prove an indirect relationship between inflation and unemployment, using the statistical data related to the US between 1915 and 1925. His studies demonstrated that the relationship was one-way-from inflation to unemployment. Phillips (1958) used the data related to 1861 to 1957 in Britain and concluded that there was an indirect relationship between the two variables. In a study in Britain between 1862 and 1957, Lipsey (1960) confirmed Phillips' findings, using the traditional theory of market behaviour. According to this theory, in case of excess demand, prices would rise and in excess supply, prices would go down. Moreover, the more markets get distant from equilibrium, the more remarkable would be the change rate. Increase in the excess demand for labour would lead to a rise in wages. Also, with increase in excess demand for labour, more unemployed individuals would undertake some financial jobs; that is to say, there is a negative relationship between unemployment and excess demand. The difference between Lipsey's (1960) and Phillips' (1958) studies is that the former addressed the relationship between the variations of the above-mentioned variables rather than the relationship between unemployment rate and wage rate. The results indicated that there was a non-linear relationship between variations in money wage rate and variations in unemployment rate, which is in step with Phillips' findings.

Lipsey (1960) rejected Phillips' (1958) hypothesis regarding threshold effect on unemployment rate-threshold effect being the amount of critical wage for affording life, less than which would make life difficult for workers. To make up for the expenses beyond the wages, workers attempt to raise their wages by bargaining through unions. Lipsey (1960) explained that Phillips curve was obtained through the horizontal addition of Phillips curves in the markets of different industries and that adopting a policy which causes change in the distribution of unemployment rate in individual markets would lead to the total shift of the curve.

As was mentioned before, the early Phillips curve expressed the relationship between unemployment and wage inflations. Policy-makers, however, determine the objectives of inflation, more often than not, in terms of change rates in prices rather than in wages.

Consequently, in order for the analysis based on Phillips curve to be useful for policy-makers' objective, it is necessary to transform it to a price change relation. Samuelson and Solow (1960) were the first to adopt such an approach. Their presupposition was that agencies determined the sales prices through a stable rule (average production cost) in which the prices are set by unit labour cost plus profit margin.

$$P_t = (1+\alpha)\frac{W_t N_t}{Y_t} \tag{1}$$

Where P_t indicates the price levels, W_t the nominal wage rate, N_t employment level and Y_t real production level. The following is obtained:

$$\dot{P} = \dot{W} - \dot{\lambda} \tag{2}$$

Where:

$\dot{\lambda}$ is the rate of labour productivity growth. Consequently, (2) shows the possibility of non-inflation increase of nominal wages, given the increase in productivity.

Samuelson and Solow (1960) introduced Phillips curve as an indicator of a trade-off between inflation and unemployment. They were the first to show the relationship between inflation and unemployment like this:

$$\pi = \pi^e + bU^{-1} - (1-\beta)\lambda \tag{3}$$

Where (π) is inflation rate, π^e predicted inflation rate, U^{-1} demand pressure in labour market and λ in $(1-\beta)\lambda$ indicates the productivity rate of labour.

These researchers demonstrated that policy-makers adapt to monetary policies to achieve various combinations of unemployment and inflation. Each point on the curve can be considered an achievable policy-making objective and the choice of this point on curve depends on the estimate of unemployment expense and inflation. Policy-makers can opt for low unemployment-high inflation combination or high unemployment-low inflation one. That is to say, substitution between inflation and unemployment is feasible.

They found out that trade-off between inflation and unemployment is not stable in long term; moreover, there is room for improving this trade-off process. Policies like retraining, establishing employment banks, etc. may add to the efficiency of labour market and lead to the displacement of the curve in a way that the rise in wage would cause a fall in unemployment.

In the 1960s, the concept of short-term Phillips curve was developed by Friedman (1968) and Phillips (1968) on the basis of expectations. In fact, two basic points were the heart and soul of Friedman's monetary school:

The first point was natural rate of unemployment obtained through combining expectations in the short-term Phillips curve. From the perspective of monetary school, there is not a permanent relationship between inflation and unemployment. They considered the short-term Phillips curve as emitting from the presupposition that labour market was continuously in a state of balance.

The second point was that there was an extraordinary emphasis on the role of money on economy. For Friedman, the effects of changes caused by the volume of money in short and long term were totally different. That is to say, changes in money supply could affect the real variables in economy and bring about expansionary effects in short term. On the contrary the rise in the volume of money, in long term, could only have influence on inflation without the ability to remarkably affect production.

One of the steps in developing Phillips's equation was the adding of expectations to the early Phillips equation by Friedman and Phelps. Friedman and Phelps challenged the early Phillips's equation simultaneously yet independent of each other, believing that it could not clearly explain the facts. They, therefore, increased its adaptability by adding expectations to it.

The objective Phelps (1968) was following in the late 1960s was to express the relationship between inflation and unemployment by pricing behaviour and wages. He placed the expectations of agents at the top of his analysis, distinguished between expected and non-expected inflation and addressed macroeconomic interpretations of this distinction. His refashioning of the curve came to be known as Phillips' generalized expectation curve. As opposed to the previous studies like Lipsey (1960), Phillips emphasized that it was the difference between expected inflation and real inflation that was related to unemployment. His analysis differed from the earlier views which regarded higher employment as possible through inflationary supply policies.

What is important for Friedman is not inflation rate but the predicted inflation (expected inflation rate). There is not a stable and permanent trade-off between inflation and unemployment. There is a natural rate of unemployment (U_N) which is compatible with the real expectations and the main forces in economy. Unemployment can only rank below inflation if the former intensifies the latter. Or it can rank above it through creating anti-inflation.

Friedman (1968) and Phelps (1968) argued separately that Phillips curve, as a trade-off process between unemployment and inflation, is not a stable and long-term equation to be capitalized on by policy-makers. They argued that both workers in labour supply and agencies in labour demand would consider the real wages and that employment would be activated when a sudden inflation occurs which escapes the notice of workers. Yet in long term, during which the sudden inflations fade away, expectations are offset by the experience of the current inflation (comparative expectations) and unemployment returns to its balance rate-a rate compatible with all inflation rates fully predicted in the stable state, indicating that the long-term Phillips curve is vertical in the natural rate of unemployment. They believe that Phillips curve moves in time and shifts to the top and the right due to the rise in the rate of expected inflation. Under inflation, unemployment would rise and under increasing unemployment inflation would rise steeply.

Combining Phillips curve and expectations with the process of learning on the basis of error, the well-known acceleration hypothesis was developed in the 1970s and caught much of policy-makers' attention. The hypothesis, which is a corollary of natural rate of unemployment, expresses that since there is not a long-term trade-off between inflation and unemployment, all attempts at fixing unemployment on a level lower than natural one would lead to a permanent risen inflation. That is due to the fact that price acceleration is so that the real inflation is more than the expected one, provided that monetary expansion is supported. Such a monetary expansion upholds the sudden inflations and prevents unemployment from returning to its balanced level. In long term, Phillips's curve is vertical in a specific natural rate, i.e., in the natural rate of unemployment. Acceleration hypothesis or Phillips curve balanced on the basis of expectations is widely accepted by economists but has not become popular yet. A great many number of economists, however, have accepted the difference between the long and short term curves but still believe that the long-term variable has a negative and steeper slope than the short-term one.

Sargent and Wallace (1975) indicated that monetary policy did not have anything to do with production procedure and employment. They utilized the rational expectation theory about the natural rate of unemployment balanced by Friedman and Phelps; the results proved that expected inflation could not have an effect on employment but transitory and unexpected inflation could reduce unemployment to a level lower than the natural rate of unemployment. They believed that systematic monetary policy-making could only affect the expected inflation and could not affect unexpected unemployment and inflation. The supporters of this school, therefore, do believe in the existence of vertical Phillips curve in short and long terms: they do not believe in the relationship between inflation and unemployment unless some deviations in production and employment from their natural levels occur due to expectation errors.

The New Classical School's attitude toward employment and unemployment was that these issues concerned the decisions of individuals: decision-takers make predictions about wages on the basis of wage in the present and determine their labour supply accordingly (Greenwald and Stiglitz, 1987). Given that these predictions might turn out to be wrong, the creation of fluctuation in employment and unemployment becomes its natural outcome.

New Keynesians, not unlike Keynes himself, began from this point that unemployment and economic fluctuations are the most pivotal matters in economy. They placed an accent on the foundations of microeconomics and believed in the rational expectations assumption but not in the clarity of new classical market. On the contrary, new Keynesians argue that at least in short term, wages and unemployment do not get balanced but, on the contrary, the contractual arrangement in the employment market keeps wages fixed for a certain time span. Consequently, the wage balance stops in the course of contract, which might stymie private sector’s balancing power in face of random tensions. If the state is able to balance its policies in facing random tensions more quickly than private sector (which can do this by renegotiating workers’ contract), there can be circumstances for implementing the policies of demand side. For the new Keynesians, unemployment is mostly unintentional; the argument of this group is stronger than new classics when it comes to explaining unemployment. In a nutshell, the assumption regarding balance in labour market espoused by New Classics has caused them to move away from their object of analysis and to be finally unable to explain unemployment. New Keynesians believe that fixing the contractual wages is a key phenomenon in the real world-which should be considered in developing a model. Furthermore, they believe in the existence of relationship between inflation and unemployment in short term-a relationship that has emerged from fixing the contractual wages. Yet these work contracts, if assumed rational, cannot be true for long term. In short term, therefore, the falling Phillips curve would be steeper than Phillips curve with Keynesian assumptions. Yet, the curve would not be vertical in short term as opposed to New Classics; it is due to lack of clarity in market. Nevertheless, in long term, it tends to be the vertical Phillips curve like other economic schools.

New Keynesians (Blanchard and Jordi (2007); Clarida et al., (1999)) have argued in rectifying traditional Keynesians’ theories that in real world there are some degrees of monetary illusion which makes it impossible for wage earners to have a complete balance in relation to inflationary expectations. In other words, expected inflation rate is taken into account in the process of determining wages albeit partially because:

1. Agencies automatically refuse to raise the wages.
2. Workers are not fully aware of situation in labour market and of the state of labour supply and demand.
3. Workers are not able to predict the future inflation rate exactly.

According to what is said above, Phillips curve will be:

$$P = \theta P^{e} + F(u), 0 < \theta < 1 \quad (4)$$

New Keynesians consider expected inflation rate to be dependent on prices in the past. In long term, then, it looks like Friedman's model $P = P^{e}$. So:

$$P = \frac{1}{1-\theta} F(U) \quad (5)$$

According to the equation above, there is a trade-off between unemployment and fully predicted inflation. In other words, long-term Phillips curve has a negative steep but is steeped in comparison to short-term curve.

2.2 Some Empirical Evidence

On one hand, the amount of literature on Phillips curve is so much that giving a comprehensive review here is not possible; on the other hand, the results of these studies are so variegated that reaching a consensus seems to be out of reach. There are a few influential studies which could be categorized as pioneer in macro-economics. This section is a brief overview of a number of recent studies conducted between years 1985 and 2009, which are relevant to our discussion.

Benderly and Zwick (1985) investigated the relationship between inflation and unemployment in the United States between 1955 and 1982. Unlike Modigliani (1977), they demonstrated in their research that variables of unemployment rate and money growth rate are the effective factors in explaining inflation rate. Grubb (1986) estimated the pattern of natural rate of unemployment for 1952-1983 time spans for 19 countries of OECD. The most outstanding result of this research is that inflation rate is not only affected by inflation rate but also by the natural rate of unemployment. Apel and Jansson (1997) investigated the relationship between inflation and unemployment and came up with an equation system for estimating potential production and unemployment rate compatible with non-accelerating inflation rate of unemployment (NAIRU).

Debell and Vickery (1998) used the seasonal data for 1959-1997 time span and investigated Phillips curve for Australia. This research estimates the patterns of short-term Phillips curve in linear and non-linear form. The function obtained from this research indicates the reverse relationship between the variables above. Gomez and Julio (2000) estimated Phillips curve for Colombia using Kalman filter and OLS. The results implied that 1 percent fall in unemployment rate would cause a rise in the predicted extent of inflation. Guerrero and Milion (2004) studied the relationship between inflation and unemployment, using rational expectations. The study draws on Markov-Switchingpattern. Even though the results of this research demonstrated the vertical Phillips curve for at least twenty years, the curve had a negative slope in the era of economic instability (1973-1983). Scheibe and Vines (2005) modelled inflation in China, using output gap Phillips curve, also known as the difference between potential and actual outputs. They estimated the retrospective as well as prospective curve. Retrospective, traditional Phillips curve was a useful point of departure for modelling inflation in China. The prospective curve, however, proved to be more appropriate than the retrospective one. The researchers arrived at the long-term Phillips curve, using the data related to 1988-2002. The results indicated that output gap, foreign exchange price and expected inflation played an important role in explaining inflation. Paul (2009) investigated Phillips curve for India. Drawing on OLS and given the industrial sector, he undertook the short-term trade-off between inflation and output gap.

3 Methodology

The statistical data for this study include the time series of 1971-2009. Kalman (1960) filter was used to investigate the substitution relation between inflation and unemployment. One of the most important applications of Kalman filter is to update the current value of ξ_t on the basis of Y_t observations. All in all, in this method, the updated figures can be obtained by having the values of F, Q, A, H or R. Another application of Kalman filter is in those equations with random variable coefficient in time. Using OLS in a regression equation estimated on the basis of time series observations, one can only come up with an estimate of model's parameters in time. As is discussed in economic matters, variables are subject to change and oscillation in time. Kalman (1960) presents a method by applying which in regression models a vector of variable coefficients in time can be obtained. In every time period in this method, the data related to the previous periods as well as the new information are used for estimating the coefficient of the next stage. The model with variable coefficient estimated by Kalman filter is a non-linear function of X_t. That is to say, even though optimal answers could be reached (in case, of course, they are distributed normally); it cannot be a linear picture of ξ_t on Y_t with abnormal interfering component. In the literature on econometrics, Kalman filter is considered an algorithm which continuously updates the information pertinent to equations in a system. In addition to the advantages this method has, it makes it possible to accurately predict matters regarding limited samples. Furthermore, an accurate likelihood function can be planned for ARMA and, finally, the estimates of VAR models can be obtained by variable coefficients in time.

In this method, if Υ_t is the coefficient vector in time, the dynamic models of V_t in terms of unobservable variables of ξ vector (rx1) which is state vector can be shown thus:

$$\xi_{t+1} = F\xi_t + V_{t+1} \tag{6}$$

$$Y_t = \acute{A}X_t + H'\xi_t + W_t \tag{7}$$

Where F, Á and H′ are the matrixes of (rxr), (nxk) and (nxr) respectively. X_t is the (kx1) vector combined of exogenous and predetermined variables. 6 is called state equation and 7 is called observation equation. It is assumed that V_t vector whose dimension is (nx1) are vectors without serial correlation.

$$E(V_t V_t') = \begin{cases} Q, t = \tau \\ 0, Other Po\text{int}\, s \end{cases} \tag{8}$$

$$E(W_t W_t') = \begin{cases} Q, t = \tau \\ 0, Other Po\text{int}\, s \end{cases} \tag{9}$$

Here, Q and R are the matrixes of (rxr) and (nxn), respectively. We have the following for τ and t:

$$E(V_t W_t') = 0 \tag{10}$$

Since X_t is a vector for exogenous variables which are predetermined, it does not include ξ_{t+s} or W_{t+s} for s=1, 2, … values. X_t, for example, may include inhibited values of variable or variables dependent on and for all values of. It is assumed here that are not correlated. That follows:

$$E(V_t \xi_1') = 0; t = 1,2,\ldots,T \tag{11}$$

$$E(W_t \xi_1') = 0; t = 1,2,\ldots,T \tag{12}$$

In most cases, X1, X2, ..., XT, ϒ1, ϒ2,..., ϒt are observable for an economic analyst. Thus, one of the most important objectives of econometric techniques is to determine the values of unobservable variables in a system on the basis of observations on hand. According to this, the numerical values of F, Q, Á, H′ and R are observable and estimable.

Kalman filter has many applications in economy-specifically in econometrics. The first and most important application may be to calculate the least squares of prediction in state variable on the basis of observations.

$$(\xi_{t+1}|t) \equiv E(\xi_{t+1}|\gamma_1) \tag{13}$$

$$(\xi_{t+1}|\gamma_1) and Y_t \equiv (\gamma', \gamma_{t-1}', \ldots, \gamma_1', X_t', X_{t-1}', \ldots, X_1' \tag{14}$$

(14) Indicates ξ_{t+1} linear picture on and is the fixed component of it. Kalman filter can steadily calculate the values of these predictions and estimate the values of least mean squared errors. Kalman's calculation method is retrogressive, which begins with values of $\xi_1|0$. The predicted value of ξ_t obtained on this basis does not contain an observation of Y on X. This is, in fact, the non-conditional mean of ξ_t.

$$\xi_1|0 = E(\xi_1) \tag{15}$$

And MSE is:

$$P_1|0 = E\{[\xi_1 - E(\xi_1)][\xi_1 - E(\xi_1)]'\} \tag{16}$$

In general, if all vectors of F unitary root are located within the circle, the process on whose basis ξ_t is formed will have a stable covariance. Thus, the mean unconditional ξ_t can be obtained by mathematic expectation via this equation:

$$\xi_{t+1} = F * \xi_t + V_{t+1} \tag{17}$$

$$E(\xi_{t+1}) = F * E(\xi_t) \tag{18}$$

Since ξ_t possesses some features of stable covariance, we will have:

$$(I_r - F) * E(\xi_t) = 0 \tag{19}$$

Since 1 cannot be a component of unitary root, (I_r-F) will be regular and the equation will have the common solution E (ξ_t=0). Unconditional variance ξ can be obtained by multiplying (17) by its transpose and then obtaining mathematical expectations from the two sides of the equation.

$$E(\xi_{t+1} * \xi'_{t+1}) + E[(F * \xi_t + V_{t+1})(\xi'_t * F' + V'_{t+1})] = F * E(\xi_t * \xi'_t) * F' + E(V_{t+1} * V'_{t+1}) \tag{20}$$

If the matrix of ξ covariance is shown with $\sum$, equation 20 will be:

$$\Sigma = F \Sigma F' + Q \tag{21}$$

This is solved thus:

$$Vec(\Sigma) = [I_r^2 - (F \otimes F)]^{-1}.Vec(Q) \tag{22}$$

In general, assuming that roots are located within the unitary circle, Kalman repetition method can begin with these primary values $(\xi_1|0)$ =0 and $(P_1|0)$ =0, which include the following relationship:

$$Vec(P_1|0) = [I_r^2 - (F \otimes F)]^{-1}.Vec(Q) \tag{23}$$

Now if some of the unitary roots of F fall outside, or on, the circle or if the primary values of state variable have not been obtained in the course of previous equations, the primary values of $(\xi_1|0)$ can be obtained on the basis of ξ_1 with the best guess. $(P_1|0)$ Matrix is a positive one which includes the extent of confidence regarding the guess about the primary value of ξ_1. Larger values in $(P_1|0)$ matrix, therefore, indicate a relatively high lack of confidence regarding the real value of ξ_1.

3.1 The Pattern

Gomez and Julio (2000) extrapolated Phillips curve for Colombia. Based on studies conducted by Laxton et al.(1999), nonlinear Phillips curve was investigated. Using Kalman filter and OLS, the results were explained. The model used was as follows:

$$\pi_t = \pi_t^e + \gamma\left(\frac{U_t^* - U_t}{U_t}\right) + \varepsilon_t^\pi \tag{24}$$

$$\pi_t^e = -\gamma + \hat{\theta}_1\pi_{t-1} + \hat{\theta}_2\pi_{t-2} + \hat{\delta}_0 S_t + \hat{\delta}_1 S_{t-1} + \hat{\delta}_2 S_{t-2} + \eta\pi_t^M \tag{25}$$

$$U_{t+1}^* = U_t^* + \varepsilon_t^U \tag{26}$$

Where π_t is the inflation rate in terms of CPI, π_t^e the inflation rate expected, U_t^* NAIRU natural rate of unemployment (see equation 26), U_t unemployment rate, S_t the index of supply shock (measured by the method presented by King and Watson, 1994) and π_t^M imported inflation.
Equation 24 can be written as follows:

$$\pi_t - \pi_t^e = \gamma\left(\frac{U_t^* - U_t}{U_t}\right) + \varepsilon_t^\pi \tag{27}$$

By simplifying the terms in parentheses, (28) will be obtained:

$$\pi_t - \pi_t^e = -\gamma + a_t Z_t + \varepsilon_t^\pi \tag{28}$$

Where $Z_t = \dfrac{1}{U_t}$ and $U^* = \dfrac{a_t}{\gamma}$.

Using (25), (26) and (28), we will have:

$$a_{t+1} = a_t + \varepsilon_t^a \tag{29}$$

$$\pi_t = a_t Z_t + [\theta_1 \theta_2 \delta_0 \delta_1 \delta_2 \eta - \gamma] \begin{bmatrix} \pi_{t-1} \\ \pi_{t-2} \\ S_t \\ S_{t-1} \\ S_{t-2} \\ \pi_t^M \\ 1 \end{bmatrix} + \varepsilon_t^{\pi} \quad (30)$$

Where $\varepsilon_t^a = \gamma \varepsilon_t^U$.

The shift equation is obviously non-static, because it is changed with time. In the state equation, Z_t coefficient changes with time as well. If variance becomes σ_a^2 =0, the value of U* will become stable. In that event, the model can be estimated by OLS method.

$$\pi_t = -\gamma + aZ_t + \theta_1 \pi_{t-1} + \theta_2 \pi_{t-2} + \delta_0 S_t + \delta_1 S_{t-1} + \delta_2 S_{t-2} + \eta \pi_t^M + \varepsilon_t^{\pi} \quad (31)$$

Where U*=a/Ɣ.

Researchers have come up with supply shock index, bringing into play the role of supply shock and using King and Watson's (1994) method:

$$S_t = 100 \times (\log P_t^A - \log P_{t-1}^A) - 100 \times (\log P_t - \log P_{t-1}) \quad (32)$$

This can be simplified thus:

$$S_t = 100 \times \log \left(\frac{\log P_t^A - \log P_{t-1}^A}{\log P_t - \log P_{t-1}} \right) \quad (33)$$

Where P_t is the inflation rate in terms of CPA and P_t^A of the inflation rate of food products. According to the theoretical principles, it is likely that in (31) the coefficient of imported inflation and the supply side shock are positive. Also, given the sign next to Z_t , which is the reverse to unemployment, it can be found out if the curve for Iran is falling or rising.

3.2 Testing Expectations in Iran

It goes without saying that changing expectations toward sensitive variables like inflation might change the primary results altogether. Because of this, economists have tried to define expectations in form of certain models so that they can make predictions. This study tests two commonly used methods in modelling expectations, i.e., adaptive expectations and rational expectations.

a. Adaptive Expectations

This type of expectations is one of the most important approaches to this issue. It is defined in terms of correcting errors in time. According to this approach, people make a decision regarding the future of a variable, using its past information. Adaptive expectations were first developed by Cagan (1956). His model was taken from first type difference linear models. It was most probably for testing and solving problems in the then common econometric methods. In this method, economic units learn from their past experiences for shaping their expectations and draw closer to the reality. The model can be written thus:

$$_{t}E(\dot{P}_{t+1})={}_{t-1}E(\dot{P}_{t})+\alpha\left[\dot{P}_{t}-E(\dot{P}_{t})\right], 0<\alpha<1 \qquad (34)$$

The equation above indicates that expectations of inflation for period t+1 are formed in period t. Hereα is a correction coefficient which indicates a proportion of expected inflation in the inflation experienced in the previous period, which is added to the model at each stage.

b. Rational Expectations

This type was first introduced by Muth (1961). He used all the information available and drew on an optimal method in the framework of microeconomics. Ten years later, it was applied by Lucas and Sargent (1979) to macroeconomics. Kara and Tuger (2005) show inflation due to rational expectations, drawing on Muth (1961):

$$\pi_t^f = E(\pi_{t+f} \mid I_t) \qquad (35)$$

Where $\pi_{t+f}$the inflation rate is in t + f time, I_t is the information available in time t and E is the mathematical operator of expectations. From (35) the following can be concluded:

$$E(\Sigma_t^f \mid I_t) = 0 \qquad (36)$$

Whereby we have:

$$\Sigma_t^f = \pi_{t+f} - \pi_t^f \qquad (37)$$

If regression analysis shows that Σ_t^f is a statistically significant function of I_t, rational expectations hypothesis (null hypothesis) can be rejected. In other words, predication cannot build an optimal situation which can be true for all information available. Obtaining all information can be an impractical and expensive process; that is why the information available is always emphasized. Bran and Mital (1981) refer to this point under partial rational expectations.

What adds to the complexity of the issue is the definition of information available or devising the expectations variable on the basis of rational expectations. To shed light on the issue at hand, testing rational expectations hypothesis is pointed out. Rational expectations hypothesis are tested in two ways as is elaborated on by Attfield et al. (1991).

First method**:** In this method, an attempt is made at modelling this type of expectations by theoretical interpretations.

$$\rho_t = E_{t-1}\rho_t + v_t \tag{38}$$

In this equation, ρ_t is the real value of index price in current period, $E_{t-1}\rho_t$ expectations pertinent to ρ_t that have been shaped in t-1 and v_t the prediction error that does not correlate with the information in t-1. Whenever $E_{t-1}\rho_t$ is observable, the following can be conjectured:

$$\rho_t = \alpha_0 + \alpha_1 E_{t-1}\rho_t + \eta_t \tag{39}$$

η_t is random error with mean 0 and variance 1. On this basis, if $\alpha_1 = 1$ and H_0: $\alpha_{0.} = 0$, in which case we cannot reject them, it can be concluded that we have accepted the rational expectations.

Second method: If ρ_t and its past values are estimated in a regression model, we will have:

$$\rho_t = \beta_1\rho_{t-1} + \beta_2\rho_{t-2} + \ldots + \beta_k\rho_{t-k} + v_{1t} \tag{40}$$

Whenever it is assumed that there are direct observations about $E_{t-1}\rho_t$, we can define $E_{t-1}\rho_t$ on the basis of past values of ρ_t as follows:

$$E_{t-1}\rho_t = \gamma_1\rho_{t-1} + \gamma_2\rho_{t-2} + \ldots + \gamma_k\rho_{t-k} + v_{2t} \tag{41}$$

Subtracting 41 from 40, we will have:

$$\rho_t - E_{t-1}\rho_t = (\beta_1 - \gamma_1)\rho_{t-1} + (\beta_2 - \gamma_2)\rho_{t-2} + \ldots + (\beta_k - \gamma_k)\rho_{t-k} + (v_{1t} - v_{2t}) \tag{42}$$

Now we can define the following hypothesis:

$$H_0 : \beta_i = \gamma_i, i = 1,2,\ldots,k \tag{43}$$

H_0 being rejected means rejecting rational expectations but in both conditions what are needed for a reliable conclusion are direct observations about the expectation variable $E_{t-1}\rho_t$ on the basis of rational expectations. Because of this, some of researchers have tried to define and substitute the above-mentioned expectation variable. Expectation variable refers to the attitude of people toward the model on whose basis we can predict the variable by using the past information, current information and the knowledge about future plans. For instance, some make these predictions on the basis of the ideas of experts, economists and businessmen and some other might use the variable of interest rate and the changes it undergoes to elaborate on the expectation variable of price index.

Since Iranian economy lacks the expectation variable to estimate (42) and also because of the role government plays in money and capital market, interest rate is not determined in a free market; determining the interest rate by the government, therefore, has caused a lack of balance in market. In practice, there is no expectation variable as such. This research resorts to a substitution variable to test rational expectations approach. In order to achieve this, variables of price index in transportation, housing and fuel are used. It seems that a remarkable lot of people are sensitive toward these indices and consider them as the inflation of coming year. Since the government has some parts of transportation and fuel in its control, it is quite rational to think that consumers in large cities—for whom transportation costs are very important—take these indices as the symbol of government's plans. As a result, these indices are used for defining $E_{t-1}\rho_t$ whereby (42) is tested. Table 1 shows the variable of transportation expenses and table 2 the price index of housing and fuels as the expectation variable.

Table 1. The Variable of Transportation Price Index

Independent Variables	Coefficient	Standard Deviation	T Coefficient	Probability Level
B_1- γ_1	-0.83	0.24	-3.39	0.018
B_2- γ_2	0.17	0.09	1.86	0.071
B_3- γ_3	0.05	0.51	0.09	0.92
B_4- γ_4	1.04	0.42	2.46	0.0193

Source: Calculations by the Authors

Table 2. The Variable of Housing and Fuel Price Index

Independent Variables	Coefficient	Standard Deviation	T Coefficient	Probability Level
B_1- γ_1	1.48	0.29	5.08	0.000
B_2- γ_2	-1.93	0.61	-3.18	0.003
B_3- γ_3	1.52	0.65	2.35	0.025
B_4- γ_4	-1.79	0.37	-4.84	0.000

Source: Calculations by the Authors

Price index of transportations, housing and fuel were considered as the substitute for the expectation variable of various inhibitions in CPI (Consumer Price Index) by using data regarding CPI. The most appropriate CPI inhibition on the basis of Akaike (1974, 1976); Schwarz (1978); Hannan and Quinn's (1979) as well as R^2 and $\overline{R}^2$ standards is four. In the next stage, given (42), the hypothesis claiming that CPI inhibition coefficients are zero was tested. If in Wald test statistic F and Chi-Square are larger than the values of their probabilities, H_0 cannot be supported. The results of Wald test and the values of F statistics and Chi-Square are summarized in Table 3.

Table 3. Wald Test

Housing and Fuel Index			Transportations Index		
Probability Level	**Value**	**Statistic t**	**Probability Level**	**Value**	**Statistic t**
0.00	161.84	Statistic F	0.00	55.091	Statistic F
0.00	647.36	Chi-Square	0.00	220.36	Chi-Square

Source: Calculations by the Authors

As can be seen here, given the values of statistic F and Chi-Square and the values of corresponding probabilities, H_0 (rational expectations) is rejected and the other hypothesis (adaptive expectations) is confirmed.

3.3 Estimating the Pattern Using Kalman Filter

Given the study conducted by Gomez and Julio (2000), the following pattern can be estimated by Kalman filter:

$$\pi_t = -\gamma + a_t Z_t + \theta_1 \pi_{t-1} + \theta_2 \pi_{t-2} + \delta_0 S_t + \delta_1 S_{t-1} + \delta_2 S_{t-2} + \eta \pi_t^M + \varepsilon_t^\pi \tag{44}$$

In this equation, Z_t coefficient is to change in time. Here are four scenarios regarding estimating Z_t:

Scenario 1:

In this scenario, Z_t coefficient is modelled in form of a stable mean in time along with interference terms.

Scenario 2:

Z_t coefficient is modelled as an AR (1) process.

Scenario 3:

Z_t coefficient is modelled as a random walk.

Scenario 4:

Z_t coefficient is modelled as a random walk process with a drift term.

4 Results

Each one of the equations obtained by the mentioned scenarios is called a transfer function. Given the supply shock, various states for state space equation can be conceptualized. Supply shock is calculated by King and Watson (1994) index. At first, the state space equation is estimated by using oil price for the four scenarios. The results are shown in tables 4 to 7.

Table 4. Estimation of Z_t coefficient on the basis of scenario 1

Name of the Variable	Coefficient	Statistic Z	Probability Level
γ	0.82	-1694.07	0.00
π_{t-1}	-3.11	20593.64	0.00
π_{t-2}	3.74	6230.25	0.00
S_t	-2.65	156666.52	0.00

S_{t-1}	-2.14	-40225.15	0.00
S_{t-2}	-2.14	-32787.78	0.00
π_t^M	-1.93	-104964.21	0.00
Sv_1	0.08	-0.08	0.93
σ_{te}^2		0.92	

Source: Calculations by the Authors

Although all coefficients in Table 4 are significant, the signs do not correspond to the theoretical foundations. For example, oil shock and its first and second suspensions have a negative effect on inflation. Also, the inflation of last year has a negative effect on inflation. In other words, the more the inflation of last year, the lesser will be the inflation in the current year. In this estimation, import inflation has an influence like the first suspension. Of all the variables, inflation with the first inhibition has a positive effect on inflation and in step with the theoretical foundations. Here what is insignificant is the transfer equation. This state, therefore, cannot be the best estimation.

Table 5. Estimation of Z_t Coefficient on the Basis of Scenario 2

Name of the Variable	Coefficient	Statistic Z	Probability Level
γ	-0.0042	-9.81e-5	0.99
π_{t-1}	-0.588	0.43	0.67
π_{t-2}	0.297	-0.28	0.77
S_t	-0.192	-1.16	0.24
S_{t-1}	-0.079	-0.33	0.75
S_{t-2}	-0.047	-0.35	0.72
π_t^M	0.027	0.74	0.45
Sv_1	83.164	1.29	0.19
AR(1)	0.014	0.009	0.99

Source: Calculations by the Authors

Given the results of scenario 2, none of the coefficients is statistically significant. Thus, no statement can be made regarding this scenario.

Table 6. Estimation of Z_t coefficient on the basis of scenario 3

Name of the Variable	Coefficient	Statistic Z	Probability Level
γ	64.41	6878.47	0.00
π_{t-1}	0.17	-8508.09	0.00
π_{t-2}	0.04	8029.12	0.00
S_t	1.04	8944.9	0.00
S_{t-1}	-0.07	-8682.24	0.00
S_{t-2}	-0.07	-8719.87	0.00
π_t^M	-0.07	-9176.24	0.00
Sv_1	614.81	-252.97	0.00
σ_{te}^2		0.34	

Source: Calculations by the Authors

As it is shown in Table 6, all coefficients are statistically significant. Inflation in the first and second inhibition has a significant and positive effect on the inflation in the current year. It can be concluded, therefore, that people shape the expectations on the basis of the inflation of the last year (adaptive expectations). Moreover, the extent of the effect of inflation in the last year on the inflation in current year is more than that of two years ago. In this estimation, the coefficient of oil shock is 1.04 which indicates that oil shock has a significant and direct effect on inflation. Of all the variables above, oil shock has the most remarkable effect. The first and second oil shock and import inflation have a reverse and meagre effect on inflation, which though significant is not in step with theoretical foundations. In this scenario, Sv_1 has turned out to be significant as well. Given its sign, which is positive, it can be concluded that the relation between inflation and unemployment is indirect. This state, therefore, can be the best estimation.

Table 7. Estimation of Z_t coefficient on the basis of scenario 4

Name of the Variable	Coefficient	Statistic Z	Probability Level
γ	0.587	0.0128	0.98
π_{t-1}	0.01	0.023	0.97
π_{t-2}	-0.22	-0.66	0.51
S_t	-0.18	-1.45	0.14
S_{t-1}	-0.16	-0.78	0.43
S_{t-2}	-0.09	-0.58	0.55
π_t^M	0.19	0.74	0.45
Sv_1	-249.53	-2.8	0.63

Source: Calculations by the Authors

In Table 7, none of the variables is statistically significant; whereby it can be concluded that the data regarding inflation and unemployment in scenarios 2 and 4 in which the coefficient of variable Z_t is modelled in the form of a AR (1) process and random walk process with a drift term do not match. Only in scenarios 1 and 3 in which Z_t (α_t) coefficient is in the form of a fixed mean in time and random walk process, the coefficients are significant. Investigating these two scenarios makes it clear that only in scenario 3 where coefficients are significant and in line with theoretical foundations, the transfer equation becomes significant.

5 Conclusion

In the course of the recent five decades, Iranian economy has gone through high inflation and unemployment rates. It is, therefore, obvious that reducing and keeping in check these two factors simultaneously are among the most important objectives of economic policy-makers and planners. The existence of a substitution relation between inflation and unemployment implies the neutralization of macro policies and of a considerable portion of facilities and resources to reduce the extent of these two factors. Given the points mentioned above and the results, it can be stated that the relationship between inflation and unemployment in Iran can be proved. Furthermore, given the reverse relationship between inflation and unemployment, it can be concluded that there is a substitution relation between inflation and unemployment in Iran. Having compared the results of a state in which supply shock is oil price with two other states, it can be concluded that oil price is very important in Iran and deserves more attention. The results of a state in which supply shock is liquidity proved that none of coefficients in four scenarios is significant. Thus, being certain regarding the parameters involved is not possible. Furthermore, when food price index was considered to be the supply shock, the only coefficients which turned out to be significant were those in scenario 2 where α_t was modelled as an AR (1) process. SV_1, however, did not prove significant in this scenario. All in all, it can be concluded that of shocks tested in this study, the only transfer equation turning out significant was the scenario 3 with oil shock. Moreover, only this scenario can state that there is a significant relationship between inflation and unemployment with substitution in Iran.

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